Wath Sixth Form Subject Preparation Pack

MATHEMATICS

World-class learning
World-class learning every lesson, every day

The highest expectations
Everyone can be successful; always set and expects the highest standards

No excuses
Create solutions not excuses; make positive thinking a habit

Growth mindset
Believe you can improve; work hard and value feedback

Never give up
Resilience is essential; be relentless in the pursuit of excellence

Everyone is valued
Diversity is celebrated; see the best in everyone
A-Level Mathematics Transition Pack

Contents

- What is Mathematics
- Why should I study Mathematics?
- What careers could A Level Mathematics lead to?
- What will I study?
- How will I be assessed?
- Recommended resources
- Additional support.

What is Mathematics?

A level Mathematics gives you the opportunity to study topics such as geometry, calculus and trigonometry (pure mathematics) and to use these ideas within the ‘applied’ topics such as mechanics and statistics. Mechanics is strongly linked to physics, and builds on ideas of motion and forces to work out how and why objects move. Statistics allows us to make sense of the complex and variable world around us via analytical methods in order to draw reliable conclusions from 'sets' of information.

Why should I study Mathematics?

Mathematics complements a whole range of other subjects and prepares you for further study and employment in many disciplines that involve the use of Mathematics. You will gain knowledge of mathematical techniques that build on GCSE knowledge and develop problem solving and analytical thinking, skills that are desirable in numerous careers. If you have enjoyed your GCSE Mathematics, especially the algebra, trigonometry and problem-solving aspects, then you should consider Mathematics at A-Level.

What careers could Mathematics lead to?

A-Level Mathematics provides a basis for subsequent quantitative work in a wide range of higher education courses and in employment. Some students go on to university to study Mathematics either as a single honours degree or in combination with another subject, for example business, science, engineering, computing, technology or humanities. Others seek employment or apprenticeships where an A-Level Mathematics qualification is valued for example accounting, engineering.

“Maths is the only A level proven to increase earnings in later life - by an average of 10%.”

(Source: www.gov.uk/government/speeches/elizabeth-truss-on-support-for-maths-and-science-teaching)
What will I study?

At Wath Academy we cover the Edexcel syllabus which covers a breadth of knowledge split across 3 papers.

<table>
<thead>
<tr>
<th>Paper 1</th>
<th>Paper 2</th>
<th>Paper 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pure Mathematics</strong></td>
<td><strong>Pure Mathematics</strong></td>
<td><strong>Statistics</strong></td>
</tr>
<tr>
<td>Algebra and functions</td>
<td>Algebra and functions</td>
<td>Representing and analysing data</td>
</tr>
<tr>
<td>Coordinate geometry and graph</td>
<td>Coordinate geometry and graph</td>
<td>Probability</td>
</tr>
<tr>
<td>Sequences and series</td>
<td>Sequences and series</td>
<td>Binomial and Normal distributions</td>
</tr>
<tr>
<td>Exponentials and logs</td>
<td>Exponentials and logs</td>
<td>Correlation and Regression</td>
</tr>
<tr>
<td>Trigonometry</td>
<td>Trigonometry</td>
<td>Hypothesis testing</td>
</tr>
<tr>
<td>Vectors</td>
<td>Vectors</td>
<td>Mechanics</td>
</tr>
<tr>
<td>Differentiation</td>
<td>Differentiation</td>
<td>Forces and motion</td>
</tr>
<tr>
<td>Integration</td>
<td>Integration</td>
<td>Kinematics</td>
</tr>
</tbody>
</table>

How will I be assessed?

Each paper above is a 2 hours examination paper, sat at the end of the course.

Recommended resources

Headstart to A-level Maths, this aims to consolidate GCSE skills in preparation for the increased demands at A-Level.

We have also compiled a booklet to help you prepare for A-Level Mathematics studies. You should have met all of the topics in the Consolidation booklet previously in GCSE. Start by working through the Consolidation booklet making sure that you check your answers as you go along. The booklet is available at the end of this pack.
Meet the A-Level Mathematics Staff

We currently have 7 members of staff that teach A-Level Mathematics. If you choose to study A-Level Mathematics at Wath Academy you will have 6 lessons a week, this will be split between two teachers. Our A-Level team are always available to help and we encourage students to ask questions and explore even further into the world of mathematics.

Mrs Casey
Head of Mathematics
Pure Maths and Statistics

Mr Billups
Pure Maths and Mechanics

Mr Marcroft
Pure Maths and Statistics

Mr Dullaway
Pure Maths and Mechanics

Miss Hodgson
Pure Maths and Statistics

Mr Dale
Further Maths

Mr Harper
Pure Maths and Mechanics

We look forward to meeting you in September!
A-LEVEL MATHEMATICS

GCSE Consolidation Pack

For A-Level Mathematics, it is essential that you are confident and competent with these underpinning skills. These skills are assumed throughout the whole of the Edexcel A-level Mathematics. Corbettmaths videos will help support you to learn any skills you are not already confident with and you can test your understanding using the practice questions and answers.

Topics

I. Pythagoras’ Theorem
II. Trigonometry
III. Expanding Brackets
IV. Factorising Quadratics
V. Simultaneous Equations – Linear & Quadratic
VI. Inequalities
VII. Indices
VIII. Surds
IX. Algebraic Fractions
X. Graphs of linear functions


Pythagoras’ Theorem

CorbettMaths Videos: 297

Practice Questions

The diagram shows a right-angled triangle.

Diagram not drawn to scale

(a) Calculate the area of the right-angled triangle. [2]

(b) Calculate the length of the hypotenuse. [3]
Diagram not drawn to scale

Calculate the length of the side marked $x$.  

[4]
The diagram shows a cross-section of the roof of a building, with the base $BCD$ being horizontal and $AC$ representing a vertical support.

![Diagram not drawn to scale]

Calculate the size of the angle that the roof $AD$ makes with the horizontal.
The sketch below shows a circle with its centre at the origin and radius 1 unit. The point \((a, b)\) is a point on the circumference of the circle.

\[(a, b)\]

\(\theta\)

(a) The angle \(\theta\) is shown on the diagram. Complete the following statements, in terms of \(a\) and \(b\).

(i) \(\sin \theta =\)

(ii) \(\tan \theta =\)

(b) Consider a general point \((x, y)\) on the circumference of the circle to write down an equation of the circle in terms of \(x\) and \(y\).
### Answers

<table>
<thead>
<tr>
<th>Question</th>
<th>Working</th>
<th>Mark Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>10. (a) $\frac{1}{2} \times 4.6 \times 2.3$ 5.29 (cm$^2$) or 5.3 (cm$^2$)</td>
<td></td>
<td>M1 A1 Only accept 5(cm$^2$) from correct working</td>
</tr>
<tr>
<td>(b) (hypotenuse$^2$) = 4.6$^2$ + 2.3$^2$</td>
<td></td>
<td>M1 A1</td>
</tr>
<tr>
<td>(hypotenuse$^2$) = 26.45 or hypotenuse = $\sqrt{26.45}$</td>
<td></td>
<td>A1 A1 Allow FT from M1, A0</td>
</tr>
<tr>
<td>7. $a^2 = 6.4^2 - 4.7^2$ $a^2 = 18.87$</td>
<td></td>
<td>M1 A1 FT their $a^2$ or their $a$ provided M1 awarded or M1 for $x$ from $\sqrt{66.82}$ to $\sqrt{77.58}$ (when $a^2 = 66.82$ or $a = 7.94$) A1 Accept 5 from correct working FT from $a^2 = 66.82$ is $x = 8.174...$ to $8.2134$</td>
</tr>
<tr>
<td>10. Strategy, using Pythagoras’ Theorem then trigonometry $8.2^2 = 6.3^2 + AC^2$ AC = $\sqrt{(8.2^2 - 6.3^2)}$ = $\sqrt{27.55}$ AC = 5.48... Tan D = AC/10.6</td>
<td></td>
<td>S1 M1 A1 Needs to involve working towards use of triangle ADC</td>
</tr>
<tr>
<td>Answers in the range 26.1...($^\circ$) to 26.33($^\circ$)</td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td>For example: S1 Strategy, using trigonometry – cos ratio, cotan ratio and sine rule $\cos B = \frac{15}{4}$ $\cos B = \frac{15}{4}$ AND $\cos B = \frac{15}{4}$ $\cos B = \frac{15}{4}$ $\cos B = \frac{15}{4}$ $\cos B = \frac{15}{4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a^2 = 66.82$ AND</td>
<td></td>
<td>M1 A1 Allow rounded or truncated answers in working throughout, but the final answer must be in the given range to award the final A1 Alternative S1 Complex strategy M1 method for both of the first 2 stages A1 for first stage answer A1 for second stage answer M1 method for the third stage A1. Answers in the range 26.1...($^\circ$) to 26.33($^\circ$)</td>
</tr>
<tr>
<td>14. (a) Idea of right angled triangle shown or used with height b, base a</td>
<td></td>
<td>M1 A1 Either answer correct implies M1</td>
</tr>
<tr>
<td>sin $\theta = b/a$</td>
<td></td>
<td>A1 A1</td>
</tr>
<tr>
<td>tan $\theta = b/a$</td>
<td></td>
<td>M1 A1</td>
</tr>
<tr>
<td>(b) Use of Pythagoras’ Theorem $x^2 + y^2 = 1$ or $x^2 = 1 - y^2$ or $y^2 = 1 - x^2$</td>
<td></td>
<td>A1 A1</td>
</tr>
<tr>
<td>M1 for $a^2 + b^2 = 1$ or $x^2 + y^2 = r^2$</td>
<td></td>
<td>A1 CO. Correct equation in any form</td>
</tr>
</tbody>
</table>
The diagram below shows a rectangle $ABCD$.

![Diagram](image)

Diagram not drawn to scale

The point $P$ is on the side $AB$.
The ratio of $AP$ to $PB$ is $3:2$.

(a) Calculate the area of the triangle $DPC$.

(b) Calculate the size of $PD$C.

(a) In the triangle below, show that $x$ satisfies the equation $3x^2 - 6x - 2 = 0$.

![Diagram](image)

Diagram not drawn to scale

(b) Solve the equation $3x^2 - 6x - 2 = 0$ and hence find the length of $BC$, correct to 1 decimal place.
From the top of a vertical cliff, the angle of depression of a sailing boat is $15^\circ$. If the sailing boat is 700 m from the base of the cliff, calculate the height of the cliff above sea level.

Diagram not drawn to scale

The area of triangle $ABC$ is 42.8 cm$^2$. Calculate the length of $AB$.

A triangular flowerbed in a park is being prepared for planting bulbs. The gardener is going to lay compost over all the flowerbed to a depth of 12 cm.

Diagram not drawn to scale

Calculate the volume of compost required.
### Answers

10. (a) \( \frac{1}{3} \times 9 \times 4 = 12 \text{ cm}^2 \)
(b) Overall strategy: use of ratio and trigonometry
\[ 9 \times 3 = 18 \]
Using angle APB = angle PDC
\[ \tan \theta = 4.5 \]
Angle PDC = \( 45^\circ \times 4 \)
\[ \tan ^{-1} 0.5 \times 2 \times 0.5 \]

11. (a) \( \frac{1}{3} \times 9 \times 4 = 12 \text{ cm}^2 \)
(b) Overall strategy: use of ratio and trigonometry
\[ 9 \times 3 = 18 \]
Using angle APB = angle PDC
\[ \tan \theta = 4.5 \]
Angle PDC = \( 45^\circ \times 4 \)

13. To be viewed with diagram.

14. Strategy: Use of \( \frac{1}{2} \) abSinC and cosine rule
\[ 42.8^\circ = \frac{1}{2} \times BC \times 17.8 \times \sin 34^\circ \]
\[ BC = \sqrt{5.99} \]
\[ AB = BC^2 + 17.8 - 2 \times BC \times 17.8 \times \cos 34^\circ \]
\[ AB = 1.36 \]

19. \( a = 4.6 \times 5.8 - 2 \times 4.5 \times 5.8 \times \cos X \) or
\[ a = 4.6^2 + 5.8^2 - 2 \times 4.5 \times 5.8 \times \cos X \]
Correct rearrangement
\[ 74.96 \ldots \] or \( 43.95 \ldots \) or \( 61.97 \ldots \)
Use of area = \( \frac{1}{2} ab \sin C \) with appropriate substitution
\[ \frac{1}{2} \times 4.6 \times 5.8 \times \sin 75^\circ \], or
\[ \frac{1}{2} \times 4.6 \times 5.8 \times \sin 44^\circ \], or
\[ \frac{1}{2} \times 4.6 \times 5.8 \times \sin 61^\circ \]
\[ 12.9 \text{ m}^2 \]
Volume compound = \( 0.12 \times 12.9 \) or \( 12 \times 12.9 \times 129.000 \)

1.5(46. m\textsuperscript{3}) or 1.346 (17.335 cm\textsuperscript{3})
Expanding Brackets

CorbettMaths Videos: 7-9, 13-14

Practice Questions

(a) Kirra needs to write a formula in a spreadsheet. She needs a formula for $g$ in terms of $f$.

Kirra knows that $f = 5 + 3g^2$.

Rearrange to make $g$ the subject of the formula.

(b) Expand and simplify $(2x + 5y)(4x - 3y)$. 
(a) Simplify \((3x + 7y)(2x - 5y) + xy\). 

\[
24x^2 - 60x + 20x - 5 \text{ AND} \\
24x^2 - 3x + 40x - 5 \text{ OR} -24x^3 + 3x - 40x + 5 \\
\text{Clearly reducing to} -6x + 20x + 3x -40x + 5 \\
\text{to} -23x \\
\text{B3 for either expansion of pair of brackets correct} \\
\text{B1 for one slip in both expansions} \\
\text{CAO: Convincing from correct working} \\
\]

Answers

<table>
<thead>
<tr>
<th>12</th>
<th>24x^2 - 60x + 20x - 5</th>
<th>AND</th>
</tr>
</thead>
<tbody>
<tr>
<td>24x^2 - 3x + 40x - 5</td>
<td>OR</td>
<td>-24x^3 + 3x - 40x + 5</td>
</tr>
<tr>
<td>Clearly reducing to</td>
<td>-6x</td>
<td>+ 20x + 3x -40x + 5</td>
</tr>
<tr>
<td>to</td>
<td>-23x</td>
<td>B3 for either expansion of pair of brackets correct</td>
</tr>
<tr>
<td>B1 for one slip in both expansions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAO: Convincing from correct working</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>7a</th>
<th>( y = 3y^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7b</td>
<td>( y = 3y^2 )</td>
</tr>
<tr>
<td>g = ( y - 2(3y^2) )</td>
<td></td>
</tr>
<tr>
<td>7c</td>
<td>( 2x(3y^2 - 2xy - 17y) = 6x^2 - 17y )</td>
</tr>
<tr>
<td>( \text{B2 for any 2 of the 3 or all correct} )</td>
<td></td>
</tr>
<tr>
<td>( \text{B1 for equivalent level of difficulty} )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>14(a)</th>
<th>( 6x^2 + 14xy - 15xy = 35x^2 + xy )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b) \</td>
<td></td>
</tr>
<tr>
<td>( 7x(3x - 2) + 2(2x + 5) ) as a numerator</td>
<td></td>
</tr>
<tr>
<td>( (2x + 3)(3x - 2) ) as a denominator</td>
<td></td>
</tr>
<tr>
<td>( 21x^2 + 14x + 4x + 6 )</td>
<td></td>
</tr>
<tr>
<td>( 21x^2 - 10x + 5 )</td>
<td></td>
</tr>
<tr>
<td>( (2x + 3)(3x - 2) )</td>
<td></td>
</tr>
<tr>
<td>( \text{B2 for any 2 of the expansion terms correct} )</td>
<td></td>
</tr>
<tr>
<td>( \text{B1 from B1. Mark final answer. Allow } 6x^2 + -35y^2 )</td>
<td></td>
</tr>
<tr>
<td>( \text{M1} )</td>
<td></td>
</tr>
<tr>
<td>( \text{M1} )</td>
<td></td>
</tr>
<tr>
<td>( \text{A1} )</td>
<td></td>
</tr>
<tr>
<td>( \text{A1} )</td>
<td></td>
</tr>
<tr>
<td>( \text{Mark final answer. If the denominator is expanded it must be correct. FT from 1 error in numerator expansion, provided it is a trinomial} )</td>
<td></td>
</tr>
</tbody>
</table>

\( \text{FT from B1} \)
Factorising Quadratics

CorbettMaths Videos: 10,118-120,266,267-267a

Practice Questions

(a) Express \(x^2 - 16x + 66\) in the form \((x + a)^2 + b\) where \(a\) and \(b\) are values to be found.

(b) Prove that \(\frac{2x + 3}{4} - \frac{3x - 2}{3} + \frac{1}{6} \equiv \frac{10 - 6x}{12}\).

(c) Simplify \(\frac{2y^2 + 5y - 7}{10y + 35}\).
(a) By factorising, solve the following quadratic equation.

\[ 8x^2 + 18x - 5 = 0 \]

(b) Use the quadratic formula to solve the following quadratic equation, giving your answers correct to 2 decimal places.

\[ 3x^2 - 5x - 7 = 0 \]

(a) Solve \( 8x - 11 = 3x + 29 \).

(b) Factorise \( 7x + 49 \).

(c) Factorise \( x^2 - 10x \).

(d) Expand \( 2x(x + 6) \).
(a) Factorise \(6x^2 + 13x - 5\).

(b) Rearrange to make \(m\) the subject of the following formula.

\[ w = \frac{3m^2}{d} \]

(c) Rearrange to make \(b\) the subject of the following formula.

\[ ab = bc + e \]
### Answers

16. (a) \((x - 8)^2 + 2\)
   
   (b) Attempt to use common denominator
   
   \[
   \frac{5(2x + 7)}{3(2x + 3)} = 2 \quad \text{or equivalent}
   \]

   \[
   \frac{5x + 9 - (3x + 2)1}{12} = \frac{10 - 6x}{12} \quad \text{with statement}
   \]

   (c) \((2x + 7)(x - 1)\)

   \[
   \frac{5(2x + 7)}{5} = \frac{x - 1}{5}
   \]

   \[
   B1 \quad \text{for } a = -8, B1 \quad \text{for } b = 1, \quad \text{only award } B2 \quad \text{for } (x - 8)^2 + 2
   \]

15. (a) \((2x + 5)(x - 1)\)

   \[
   x = \frac{-1}{2} \quad \text{AND} \quad x = 1/4
   \]

   \[
   B2 \quad B1 \quad \text{for } (2x + 5)(x - 1)
   \]

   \[
   B1 \quad \text{FT from their pair of brackets, equivalent difficulty}
   \]

   \[
   M1 \quad A1 \quad \text{No marks for the use of the quadratic formula, or trial & improvement}
   \]

15(b) \((x - 1)^2 - 4x(x - 7) + 2x^3\)

   \[
   \frac{5x + (x^2 - 4x)(x - 7)}{2x} = \frac{5x - 0.91}{6.25} \quad \text{and} \quad -0.91
   \]

   \[
   M1 \quad A1 \quad \text{CAO}
   \]

   \[
   \text{Accept method of completing square}
   \]

---

5(a) \(8x - 3x = 28 + 11\)

   \[
   5x = 40 \quad \text{OR} \quad x = 0/5
   \]

   \[
   B1 \quad B1 \quad \text{Must be simplified}
   \]

5(b) \((x + 7)\)

   \[
   B1 \quad \text{CAO}
   \]

5(c) \((x - 10)\)

   \[
   B1 \quad \text{CAO}
   \]

5(d) \(2x^2 + 12x\)

   \[
   B2 \quad \text{Must be as one complete expression.}
   \]

   \[
   \text{Mark final answer}
   \]

   \[
   B1 \quad \text{for each term}
   \]

---

12(a) \((4x + 5)(3x - 1)\)

   \[
   (b) \quad \text{rewritten} \quad \text{m} = \frac{x}{x/3} \quad m = \frac{x}{x/3}
   \]

   \[
   (c) \quad \text{Collect} \quad \text{b(a - c)} = b \quad \text{for } \text{CBM from } 1 \text{ min. error}
   \]

   \[
   B1 \quad B1 \quad \text{Factorize}
   \]

   \[
   B1 \quad \text{Divide}
   \]

---

12(b) \((2x + 5)(3x - 1)\)

   \[
   (b) \quad \text{rewritten} \quad \text{m} = \frac{x}{x/3} \quad m = \frac{x}{x/3}
   \]

   \[
   (c) \quad \text{Collect} \quad \text{b(a - c)} = b \quad \text{for } \text{CBM from } 1 \text{ min. error}
   \]

   \[
   B1 \quad B1 \quad \text{Factorize}
   \]

   \[
   B1 \quad \text{Divide}
   \]
Simultaneous Equations – Linear & Quadratic

CorbettMaths Videos: 298

Practice Questions

The diagram below shows a composite shape formed by joining two rectangles.

Diagram not drawn to scale

The area of the larger rectangle is $4y \text{ cm}^2$.
The area of the smaller rectangle is $y \text{ cm}^2$.
Form and solve simultaneous equations to calculate the dimensions of the smaller rectangle.
Give your answers correct to 1 decimal place.

(a) Solve the following simultaneous equations using an algebraic method.

\[
\begin{align*}
3x^2 + xy + 6 &= 0 \\
x + y &= 8
\end{align*}
\]

(b) Use the formula method to solve $2x^2 + 5x - 4 = 0$, giving your answer correct to 2 decimal places.

A rectangle of length $(x + 8) \text{ cm}$ and width $x \text{ cm}$ has an area of $y \text{ cm}^2$.
It is known that $y - x = 1284$.
Find the dimensions of the rectangle.
Give your answer correct to 1 decimal place.
You must use an algebraic method.
The Davies family want to buy some garden furniture. Their local garden centre stocks the particular brand they would like.

The price of one garden bench and four chairs is £310.
The price of two garden benches and three chairs is £345.

The Davies family have £450 available to spend.

Do they have enough money to buy two garden benches and five chairs?

You must show all your working.
16. \( x(x + 2) - y \) (ISW)  
\( 12(5 + x) - 4y \) (ISW)  
\( x(x^2 + 2) = 15 + 3x \) OR \( 12(5 + x) = 4x(x + 2) \) \( \text{OR equivalent} \)  
\( x^2 - x - 15 = 0 \) OR \( 4x^2 - 4x - 60 = 0 \)  
\( x = \{1 \pm \sqrt{1^2 - 4 \times (-15)}\}/2 \) \( \text{OR} \)  
\( x = \{4 \pm \sqrt{4^2 - 4 \times 4 \times (-60)}\}/2 \times 4 \)  
\( x = (1 \pm (6)/2) \) OR \( x = (4 \pm \sqrt{976})/8 \) \( (x =) 4.4 \text{ cm} \) \( (x + 2 =) 6.4 \text{ cm} \)  
| B1 | The first two B1 marks may be implied by the correct quadratic, hence if M1 awarded also award BOTH these B1 marks |  
| M1 | Equating or substituting, allow 1 error, e.g. missing brackets, or from incorrect expansion. FT provided at least B1 and equivalent level of difficulty |  
| A1 | Must be equate to zero |  
| M1 | Allow 1 slip in substitution. FT ‘their 3 term quadratic’ of equivalent level of difficulty for M1 and final A1 only |  
| A1 | \((-3.4 \text{ is not required})\) |  
| A1 | FT provided M1 awarded |  

12(a) \( y = 8 - x \) OR \( x = 8 - y \)  
\( 3x^2 + 8 - (8 - x) + 6 = 0 \) OR \( 3(y-2)^2 + 8 - y + 6 = 0 \)  
\( 2x^2 + 8x + 6 = 0 \) OR \( 2y^2 - 40y + 108 = 0 \)  
\( (x+1)(x+3) = 0 \) OR \( (y-11)(y-9) = 0 \)  
\( x = -1, x = -3 \) OR \( y = 11, y = 9 \)  
\( y = 6, y = 11 \) OR \( x = -1, x = -3 \)  
\( (b) \ x = \{4 \pm \sqrt{5^2 - 4 \times 2 \times (-4)}\}/2 \times 2 \) \( \text{OR} \)  
\( x = \{4 \pm \sqrt{57}/4 \) \( \text{OR} \)  
\( 9.64 \text{ and } -3.14 \)  
| M1 | Allow 1 error in substitution |  
| A1 | Or by halving \( x^2 + 4x + 3 = 0 \) OR \( y^2 - 20y + 99 = 0 \) |  
| M1 | FT equivalent level of difficulty provided M1 |  
| A1 | Or alternative method to solve, e.g. formula with correct substitution and \( b^2 - 4ac \) correct simplified |  
| A1 | OR \( S\)C for \( x = -1, y = 9 \) OR \( x = -3, y = 11 \) |  
| M1 | Allow 1 slip in substitution. Must be correct formula |  

15. Equation \( x(x+8) = y \)  
Sight of \( x(x+8) = y \) AND \( y = 1284 + x \), OR \( x(x+8) = x + 1284 \) \( x^2 + 7x - 1284 = 0 \)  
\( x = \{7 \pm \sqrt{49 + 4 \times 1284}\}/2 \) \( \text{OR} \)  
\( x = \{-72 \pm \sqrt{51857}/2 \) \( \text{OR} \)  
\( x = 32.5(0337... \text{ (and } -39.5... ) \) \( \text{Dimensions are } 32.5(0337... \text{ cm by } 40.5(037... \text{ cm} |  
| B2 | Allow B1 for sight of \( x(x+8) \) |  
| B1 | Must be rearranged form \( y = 1284 + x \), \( \text{FT from 1 error} \) |  
| M1 | Allow 1 slip |  
| A1 | \( \text{FT for } +ve x \) and \( x+8 \) provided M1 awarded, with answer correct to 1 d.p. |  
| A1 | Watch for alternative with elimination of \( x \) instead of \( y \), and mark accordingly |  

8
Inequalities

CorbettMaths Videos: 178-182

Practice Questions

(a) Solve \( \frac{40}{2x} = 4 \).

(b) Solve \( 3(2x - 5) \geq 21 \).

(c) Make \( g \) the subject of the formula \( 3g^2 - f = 0 \).
Complete the following table to give the set of inequalities that describes the shaded region drawn above.

<table>
<thead>
<tr>
<th>x</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>
(a) Simplify \(4(x + 5) - 3(2x - 4)\).

(b) Simplify \(\frac{y^{16} \times y^2}{y^4}\).

(c) Solve \(3b + 2 > 29\).
Using the axes below, find the region defined by the following inequalities.

\[
\begin{align*}
    x &\geq -2 \\
    y &\geq 1 \\
    x + 2y &\leq 4
\end{align*}
\]

Make sure that you clearly indicate the region that represents your answer.
**Answers**

<table>
<thead>
<tr>
<th>4(a)</th>
<th>( \frac{d}{dx} = 5x \text{ or } 3x = 13 \text{ or } 60 = 20x \text{ or } 40 = 4 \times 5x \text{ or } 8 \times 4x )</th>
<th>M1</th>
<th>A1</th>
<th>Accept embedded answers. Mark final answer.</th>
</tr>
</thead>
</table>

| 4(b) | \( 6x - 15 > 21 \) or \( 2x - 5 > 21/3 \) | B1 | FT until 2nd error. Use of \( = \) gets no marks, unless replaced in final answer, then awarded 1 mark. |
|---|---|---|---|---|
| | \( 6x > 21 + 15 \) or \( 6x > 36 \) or \( 2x > 7 + 5 \) or \( x > 6 \) | B1 | Mark final answer. Must be a simplified answer. |
| | | B1 | If working shows \( 6x - 15 > 21 \), then \( 6x > 6 \) and then \( x > 1 \) award B1, B0, B1 |
| | | | 2x - 5 > 21 is 2 errors. Hence B0 |

<table>
<thead>
<tr>
<th>4(c)</th>
<th>( 3g^2 = f )</th>
<th>B1</th>
<th>Accept ( 3g^2 = f + 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( g = \sqrt{3} )</td>
<td>B1</td>
<td>Accept inclusion of ( \sqrt{3} )</td>
</tr>
<tr>
<td></td>
<td>( g = \pm \sqrt{3} ) (if ( f(3) ) )</td>
<td>B1</td>
<td>Square root clearly needs to include the ( \pm ), and accept inclusion of ( \pm )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Do not accept ( \sqrt{3} ) instead of ( 0 + f )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( 9g^2 = f ) is 1 error, first B0, then FT</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>7</th>
<th>( y \geq -1 ) or equivalent ( y \leq 2x )</th>
<th>B1</th>
<th>Accept ( &gt; )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( y \leq 2x )</td>
<td>B2</td>
<td>Accept ( &lt; )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1 ( 2x, y \leq 2x ) with ( k &gt; 0 )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5(a)</th>
<th>( 6x + 20 - 6x + 32 = -2n + 32 )</th>
<th>B1</th>
<th>FT until 2nd error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>B1</td>
<td>Mark final answer.</td>
</tr>
</tbody>
</table>

| 5(b) \( y^2 \) | B1 |

| 5(c) | \( 3b > 27 \) | B1 | \( b > 9 \) |

<table>
<thead>
<tr>
<th>11</th>
<th><strong>Overlay required</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Night of line for either ( x = -2 ) or ( y = 1 )</td>
</tr>
<tr>
<td></td>
<td>Correct line drawn for ( x + 2y = 4 )</td>
</tr>
</tbody>
</table>

| | B1 | Accept an unlabeled correct line provided unambiguous. Accept dotted lines throughout question. |
|---|---|---|---|
| | B2 | BI for correct gradient \( = -\frac{1}{2} \) |
| | | OK correct y-intercept plotted (0, 3) |
| | | OK correct x-intercept plotted \( (4, 0) \) |
| | | OK any two other points calculated or plotted correctly (with no incorrect points) |
| | | B1 | FT for their lines (for equivalent difficulty). |
Indices

CorbettMaths Videos: 172-175

Practice Questions

Raul has been asked to look at some data. He is asked to write the data in the form $2^n$, where $n$ is a whole number or a decimal.

Write the following numbers in the form $2^n$.

(a) $\frac{1}{2^3}$

(b) $(2^3)^{0.4}$

(c) $(\sqrt[3]{8})^{12}$

(a) Express $8^{\frac{1}{3}}$ as a decimal.
(a) Express $1000^{\frac{2}{3}}$ as a decimal.

(b) Simplify $\sqrt{2^3 \times \sqrt{64}}$.

[3]
## Answers

<table>
<thead>
<tr>
<th>Question</th>
<th>Marks</th>
<th>Notes</th>
</tr>
</thead>
</table>
| 10 (a) $2^3$  
(b) $2^{11}$  
(c) $2^7$ | B1 | Accept $2^{11/100}$ or equivalent. B1 for sight of $8^{1/2}$ or $8^{1/4}x^2$ or $8^4$ or $(2^3)^{1/4}$ or equivalent. If a candidate writes only the index, penalise -1 once only on the first occasion. |
| 13 (a) $0.25$  
(b) $4(x - 20)(x + 20)$  
(c) $27$  
(d) $10 + \sqrt{5/2} - 2\sqrt{2} - 6$ | B2 | B1 for 2 correct steps, following through 1 error: reciprocal, cube root, square. B1 for an answer of 1/4 B1 for $4(x - 20)(x + 20)$ or $2(x - 40)(2x - 40)$ |
| 14 (a) $1/1000^2$ or $10^{-4}$ or similar first step  
1/100 or 1/1000 or similar second step  
0.01 | M1 | Needs reciprocal. First M1 mark may be implied. CAO. |
| (b)(i) $a = 7$  
$x^2 + 14x + 49$ OR method to find $b$  
b = -2 | M1 | Accept embedded answers. A1 |
| (ii) $(x + 7)^2 = 9$  
$(x + 7) = 2$  
$x + 7 = \pm\sqrt{2}$  
x = $\pm\sqrt{2} - 7$ | B1 | Accept embedded answers. Use of completing the from (i) |
| 6 (a) $15$  
(b) $2\times3$ and $3\times7$ OR other method  
42 | B1 | CAO. Or sight of $2\times3\times7$ |
| (c) $\sqrt{(8 \times 8)}$  
8 | A1 | CAO |
Surds

CorbettMaths Videos: 305-308

Practice Questions

(c) Evaluate $(\sqrt{3})^6$. 


(d) Simplify $(2 + 3\sqrt{2})(5 - \sqrt{2})$. 


(b) Simplify $(\pi\sqrt{5} - \pi\sqrt{5})^2$, leaving your answer in terms of $\pi$. 


(a) Evaluate $19^9$. 


(b) Find the value of $(\sqrt{80} - \sqrt{5})^2$. 


(a) Evaluate $6\sqrt{5} \times 2\sqrt{5}$.

(b) Evaluate $(7\sqrt{2} - 4\sqrt{2})^4$.

(c) Simplify $\sqrt{288}$.
Write your answer in surd form.
### Answers

<table>
<thead>
<tr>
<th>Question</th>
<th>Mark Scheme</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>12(a) 1</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>12(b) (\sqrt{80} = \sqrt{4 \times 5} ) or (\sqrt{4 \times 5 + 5} )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td>CAO</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15(a) 60</td>
<td>B2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1 for sight of (\sqrt{5} \times \sqrt{5} = 5 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B2 for (3+2)^2 with an attempt to evaluate, B1× or ×4, OR 18×18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1 for (3×2)^2, OR multiply pair brackets to 18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c) (12\sqrt{2} )</td>
<td>B2</td>
<td>Mark final answer</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1 for sight of 258 broken down into factors e.g. 2×144, or 2(28^2) = 144, or 12×12=2, or 4×72, or 2(12^2) = 144, or 2^3×72, or 2^3×6^2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**CAO:** Mark final answer.
Algebraic Fractions

CorbettMaths Videos: 21-24, 111-111a

Practice Questions

(b) Prove that \( \frac{2x + 3}{4} - \frac{3x - 2}{3} + \frac{1}{6} = \frac{19 - 6x}{12} \).

Solve \( \frac{8x - 5}{3} + \frac{4x + 5}{4} = \frac{149}{12} \).
Solve the following equation.

\[ \frac{5x - 1}{2} - x = \frac{1}{2} \]

Solve the equation \( \frac{4x - 5}{2} + \frac{7x - 10}{5} = 4 \).
(a) Simplify \( \frac{x^2 - 81}{2x^2 + 13x - 45} \).

(b) Express \( \frac{7x}{2x + 3} + \frac{2}{3x - 2} \) as a single fraction in its simplest form. [4]
### Answers

9. \[ 4(8x - 5) + 3(4x + 5) = 149 \]
   
   \[ 44x - 5 = 149 \]
   
   \[ x = \frac{154}{44} (= 3.5) \]

- **M2**: For correctly clearing all 3 fractions, OR
  - **M1**: for clearing 2 fractions
  - **A1**: FT from M1 (for all further marks), for their equation, for correct expansion of bracket(s) and collection of like terms on LHS
  - **A1**: Ignore incorrect cancelling

9. \[ 5x - 1 - 2x = 1 \]
   
   \[ 3x = 2 \]
   
   \[ x = \frac{2}{3} \text{ or equivalent} \]

- **B1**: Clearing fractions. FT until 2nd error.
- **B1**: Collecting terms.
- **B1**: FT \( ax = b \), with \( a \neq 1 \)

### Unitised Unit 3 – Nov 2015

**Higher Tier**

<table>
<thead>
<tr>
<th>Question</th>
<th><strong>FINAL MARK SCHEME</strong></th>
</tr>
</thead>
</table>
| **Comments** | **M2** M1 for correctly applying a consistent multiplier to 2 of the terms to clear the fractions.  
Mark their final answer.  
If no marks awarded, SC1 for sight of (34x - 45)/10. | **A1** FT from M1.  
**A1** Mark their final answer.  
4 If no marks awarded, SC1 for sight of (34x - 45)/10. |
| 11. \[ 5(4x - 5) + 2(7x - 10) = 40 \text{ or equivalent} \]  
\[ 34x - 45 = 40 \text{ or equivalent} \]  
\[ x = 2.5 \] | **B1** Clearing fractions. FT until 2nd error.  
**B1** Collecting terms.  
**B1** FT \( ax = b \), with \( a \neq 1 \) |
| 18. (a) \[ (x + 9)(x - 9) \]  
\[ (2x - 5)(x + 9) \]  
\[ x = \frac{9}{2} \]  
\[ 2x - 5 \] | **B1** for \( (2x - 5)(x + 9) \)  
**B2** FT if possible for similar level of difficulty  
Mark final answer, do not ISW |
| (b)(i) \[ a = 6 \]  
\[ x^2 + 12x + 36 \text{ OR method to find } b \]  
\[ b = -22 \]  
(ii) \[ (x + 6)^2 = 22 \]  
\[ (x + 6)^2 = 22 \]  
\[ x + 6 = \pm \sqrt{22} \]  
\[ x = \pm (22 - 6) \] | **B1** Accept embedded answers  
**B1** Use of completing the square from (i), FT  
**M1** Mark final answer.  
11 Must show \( \pm \) |
| (b) \[ 7(3x - 2) + 2(2x + 3) \text{ as a numerator} \]  
\[ (2x + 3)(3x - 2) \text{ as a denominator} \]  
\[ 21x^2 - 14x + 4x + 6 \]  
\[ \frac{21x^2 - 10x + 6}{(2x + 3)(3x - 2)} \] | **M1** Mark final answer.  
**A1** If the denominator is expanded it must be correct. FT from 1 error in numerator expansion, provided it is a trinomial |
Graphs of linear functions

CorbettMaths Videos: 185-198

Practice Questions

In an experiment, the weight added to the end of a vertical spring is gradually increased.

Diagram not drawn to scale

At the end of the experiment, a computer produced the graph shown below.

Length of spring (mm)

(a) Write down the length of the spring without any weight added.

[2]
(b)  (i) Calculate the gradient of the straight line drawn on the graph.

(ii) Explain what the gradient of this graph tells you in relation to the experiment.

(c) The straight line stops before the right-hand edge of the graph paper. Why do you think this might be?

(d) In a similar experiment, weights of up to 100 g are added to the end of a different spring. The following data is recorded.

<table>
<thead>
<tr>
<th>Weight added, ( W ) (grams)</th>
<th>0</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of spring, ( L ) (mm)</td>
<td>35</td>
<td>235</td>
</tr>
</tbody>
</table>

Using the data given above, complete the formula to give the length of the spring, \( L \) mm, for different weights added, \( W \) g.

\[ L = \ldots W + \ldots \]
<table>
<thead>
<tr>
<th>Line</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$y = 3x + 4$</td>
</tr>
<tr>
<td>B</td>
<td>$y = -3x + 3$</td>
</tr>
<tr>
<td>C</td>
<td>$y = -2x - 4$</td>
</tr>
<tr>
<td>D</td>
<td>$y = 3x - 5$</td>
</tr>
<tr>
<td>E</td>
<td>$y = 4x + 4$</td>
</tr>
</tbody>
</table>

(a) Which two of the above lines are parallel? You must give a clear reason for your answer.

(b) Which two of the above lines intersect each other on the $y$-axis?
(a) Find the equation of the straight line shown in the following diagram. Write your answer in the form $y = mx + c$. [2]

Equation of the straight line is $y = \_\_\_\_\_\_\_\_\_\_ \times + \_\_\_\_\_\_\_\_\_\_$. 

(b) On the grid above, draw the straight line which has a gradient of $-2$ and which passes through the point $(0, -1)$. [2]
The table shows some of the values of \( y = x^3 + 6 \) for values of \( x \) from \(-2\) to \(3\).

(a) Complete the table by finding the value of \( y \) for \( x = -1 \) and \( x = 2 \). [2]

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
</table>
| \( y = x^3 + 6 \) | \(-2\) | \(6\) | \(7\) | \(33\) |}

(b) On the graph paper below, draw the graph of \( y = x^3 + 6 \) for values of \( x \) from \(-2\) to \(3\). [2]

(c) Faye wants to solve the equation \( x^3 + 6 = 10 \) by first drawing a line on the graph above. Show how Faye would do this on the graph above. You do not need to find the solution of the equation. [1]
<table>
<thead>
<tr>
<th>Question</th>
<th>Mark Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>4(a) 8 mm</td>
<td>B2: B1 for an answer of 8, or for an answer between 6 and 9 inclusive (excluding 8) with mm given. Mark final answer.</td>
</tr>
<tr>
<td></td>
<td>M1: Or idea of alternative complete method.</td>
</tr>
<tr>
<td></td>
<td>A1: Accept sight of quadrant based on misread of the scale for M1 only. Or alternative complete method with accurate values.</td>
</tr>
<tr>
<td></td>
<td>E1: For a complete explanation, e.g., 'rate of change length with weight', 'for every 1g increase 0.08mm increase', 'broken spring', 'spring now completely straight', etc.</td>
</tr>
<tr>
<td></td>
<td>E2: For a mention of 'rate of change' without being specific, e.g., '1g gives 0.08mm'.</td>
</tr>
<tr>
<td></td>
<td>E3: Allow 'length increases as weight increases' for E1 only.</td>
</tr>
<tr>
<td></td>
<td>B3: B2 for either of 2 or 35 correctly placed, or B1 for a correct interpretation of data given, e.g., sketch shows start (0,35) and end (10, 235).</td>
</tr>
<tr>
<td>10. (a) A and D selected</td>
<td>B1: Depends on B1 being awarded. Accept 'slope' or similar.</td>
</tr>
<tr>
<td></td>
<td>E1: m = 3 or 'both 3x' is insufficient, needs interpretation.</td>
</tr>
<tr>
<td></td>
<td>A and E selected</td>
</tr>
<tr>
<td>7(a) y = 3x + 6</td>
<td>B2: B1 for y = 3/2 x + 6 or equivalent, or for m = 3 (must be clear that this is the gradient), or for c = 6 (must be clear that this is the intercept).</td>
</tr>
<tr>
<td>7(b) Correct straight line drawn (gradient -2, passing through (0, -1))</td>
<td>B2: B1 for any straight line drawn with gradient -2, or a straight line passing through (0, -1) with either gradient 2 or with a negative gradient.</td>
</tr>
<tr>
<td>6(a) 5 and 14</td>
<td>B2: B1 for each.</td>
</tr>
<tr>
<td></td>
<td>B1: FT from (a).</td>
</tr>
<tr>
<td></td>
<td>B1: FT from (a). Need to have all 5 plots no omissions.</td>
</tr>
<tr>
<td></td>
<td>B1: Sight of y = 10 including the intersection, or marking the intersection and y = 10 with a point, or a vertical line to the point of intersection with y = 10.</td>
</tr>
<tr>
<td></td>
<td>B1: FT their graph. Unambiguous answer on the graph.</td>
</tr>
</tbody>
</table>